Current quark mass and nonzero-ness of chiral condensates in thermal Nambu-Jona-Lasinio model [*]

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Abstract

The effect that the current quark mass M_0 may result in nonzero-ness of chiral condensates is systematically reexamined and analyzed in a two-flavor Nambu-Jona-Lasinio (NJL) model which simulates Quantum Chromodynamics (QCD) at temperature T and finite quark chemical potential μ without and with electrical neutrality (EN) condition and at any T and μ without EN condition. By means of a quantitative investigation of the order parameter m indicating the chiral symmetry breaking in the model's ground states, it is shown that a nonzero M_0 is bound to lead to nonzero quark-antiquark condensates throughout chiral phase transitions within the frame of the NJL model, no matter whether the order parameter m varies discontinuously or continuously. In fact, a complete disappearance of the quark-antiquark condensates are proven to demand the non-physical and unrealistic conditions $\mu \geq$ or $\gg \sqrt{\Lambda^2 + M_0^2}$ if $T = 0$ and finite, or $T \to \infty$ if $\mu < \sqrt{\Lambda^2 + M_0^2}$, where Λ is the 3D momentum cut of the loop integrals, the largest physical mass scale in the NJL model. Theoretically these results show that when M_0 is included, besides the explicit chiral symmetry breaking indicated by M_0, different from the chiral limit case, we never have a complete restoration of dynamical (spontaneous) chiral symmetry breaking, including after a first order chiral phase transition at low T and high μ. In physical reality, it is argued that these results play a decisive role in the known phase diagram of the model. It is the nonzero-ness of the quark-antiquark condensates that leads to the appearance of a critical end point in the first order phase transition line and the crossover behavior at high T and/or high μ cases, rather than a possible tricritical point and a second order phase transition line. They also provide the basic reason for that one must consider the interplay between the chiral and diquark condensates in the research on color superconductor at zero T and high μ case. The whole discussions make us learn how a source term of the Lagrangian (at present i.e. the current quark mass term) can greatly affect dynamical behavior of a physical system.

1 INTRODUCTION

By means of the Nambu-Jona-Lasinio (NJL) model [1] which simulates Quantum Chromodynamics (QCD) , one has made extensive research on chiral phase transitions at finite temperature T and finite quark chemical potential μ (corresponding to quark matter density)[2, 3]. It is found that in the chiral limit where quarks have zero current masses,

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the dynamical (spontaneous) chiral symmetry breaking in vacuum induced by the quark-antiquark condensates, or say, chiral condensates $\langle \bar{q}q \rangle$ [1, 4, 5, 6] will be restored at high T and/or high μ, and this restoration will always be accompanied with the condensates $\langle \bar{q}q \rangle = 0$, no matter whether the symmetry restoring phase transition is first or second order [7, 8].

On the other hand , when there is a nonzero current quark mass M_0, in the vacuum we will have both the explicit chiral symmetry breaking indicated by M_0 and the dynamical chiral symmetry breaking induced by the condensates $\langle \bar{q}q \rangle \neq 0$, thus the order parameter m indicating chiral symmetry breaking will include the contributions from both M_0 and the condensates $\langle \bar{q}q \rangle$. Usually one views m as a whole order parameter indicating chiral symmetry breaking and does not distinguish it into M_0 and the contribution from the condensates $\langle \bar{q}q \rangle$. However, in the process of chiral phase transitions at high T and/or high μ, seeing that M_0 is a fixed parameter, what can actually be changed is merely the $\langle \bar{q}q \rangle$ sector. Therefore, the chiral phase transitions practically only depend on variation of the condensates $\langle \bar{q}q \rangle$, but such variation must presuppose existence of the current quark mass M_0. In the Lagrangian of the model, the current quark mass M_0 corresponds a source term whose existence, in general, will inevitably affect on dynamical behavior of the model, in present case, i.e. the change of the condensates $\langle \bar{q}q \rangle$. In view of this, it is certainly an significant topic of the change of the chiral condensates $\langle \bar{q}q \rangle$ and its effect on chiral phase transitions when a current quark mass M_0 exists. To focus our attention on this topic and to compare the obtained results with the ones in the chiral limit case where $M_0 = 0$ will be able to make us acquire a deeper theoretical insight of chiral phase transitions of the NJL model.

As stated above, in the chiral limit, the condensates $\langle \bar{q}q \rangle$ could be equal to zeros at some high T and/or high μ, hence the dynamical chiral symmetry breaking will be restored completely. Now one can ask if a similar result could appear when the current quark mass M_0 exists, i.e. at some high T and/or high μ, the condensates $\langle \bar{q}q \rangle$ disappear totally thus the dynamical (spontaneous) sector of the chiral symmetry breaking will be restored completely. If the answer is no, then what physical effects this will actually bring about?

The above questions motivate the research of the present chapter. Although there have been many work about chiral phase transitions in a NJL model with a current quark mass, it seems still to lack a systematical and deep examination of the problem from the point of view considering the relation between the current quark mass and the chiral condensates, or say, the one between the explicit and spontaneous chiral symmetry breaking. The discussions in present chapter will just focus to this respect. It should be indicated that in the discussions we will revisit the process of chiral phase transitions at different conditions and this will naturally involve, at least qualitatively and partly, some known results. However, it is essential that we will analyzes all the the results from the above specific point of view and anticipate to get a deeper understanding about the mechanism of chiral phase transitions of the NJL model.

In the following discussions , we will use the term "complete restoration of the dynamical chiral symmetry breaking" to express the condensates $\langle \bar{q}q \rangle = 0$. However, it should be remembered that in the case with the current quark mass M_0, different from the chiral limit case , the complete restoration of dynamical chiral symmetry breaking does not mean chiral symmetry restoration, because after all we always have the explicit chiral symmetry breaking indicated by M_0.

We will explore the above topic by means of a two-flavor NJL model which simulates QCD in the mean field approximation. The Lagrangian of the model can be expressed by

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - M_0)q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2] \tag{1}$$

with the quark Dirac fields q in the $SU_f(2)$ doublet and the $SU_c(3)$ triplets, i.e.

$$q = \begin{pmatrix} u_i \\ d_i \end{pmatrix}, \quad i = r, g, b \text{ (three colors)},$$

$\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices, G_S is the four-fermion coupling constants and M_0 represents the common current mass of the u and d quarks. The chiral $SU_{fL}(2) \otimes SU_{fR}(2)$ flavor symmetry of the Lagrangian (1) will be broken not only explicitly by the current quark mass M_0, but also assumedly in the vacuum spontaneously by the scalar quark-antiquark condensates $\langle \bar{q}q \rangle$ formed through the four-fermion interactions $G_S(\bar{q}q)^2$. The constituent quark mass i.e. the order parameter indicating chiral symmetry breaking will be $m = M_0 - 2G_S\langle \bar{q}q \rangle$. It is seen from this definition of m that the chiral $\langle \bar{q}q \rangle = 0$ will mean that $m = M_0$. The emphasis of research will be put on the effect of the current quark mass M_0 on changes of the chiral condensates $\langle \bar{q}q \rangle$ and the relevant physical results, including the interesting problem that if the dynamical chiral symmetry breaking could be restored completely at a high T and/or a high μ when M_0 exists.

In Sect.2 and 3, we will describe the model's chiral phase transitions in $T = 0$ and high μ case respectively without and with electric neutrality (EN) condition and compare the obtained results. By means of a quantitative analysis of the locations of the least value points of the effective potential, we will focus on the changes of the chiral condensates in the phase transitions and the comparison of the results with the ones in chiral limit case. We will also find out the conditions in which the chiral condensates could be reduced to zeros and indicate the non-physical feature of the conditions. In Sect.4, a general analysis of chiral phase transitions of the model will be conducted in any T and μ cases without EN condition. We will indicate the decisive role of nonzero-ness of the chiral condensates in the phase diagram and give a general demonstration of the nonzero-ness of the chiral condensates within the frame of the NJL model. Finally, in Sect.5 we come to our conclusion.

2 ZERO T AND HIGH μ PHASE TRANSITIONS WITHOUT ELECTRICAL NEUTRALITY

In this section, we will revisit the chiral phase transitions of the model at $T = 0$ and finite μ without the EN condition and track the variations of the order parameter indicating chiral symmetry breaking as μ increases.

When $T = 0$, in the mean field approximation, the effective potential of the model can be expressed by [9]

$$
\begin{aligned}
&V(m, \mu) \\
&= \frac{(m - M_0)^2}{4G_S} - 12 \int \frac{d^3p}{(2\pi)^3} \left[E - \sqrt{\vec{p}^2 + M_0^2} + \theta(\mu - E)(\mu - E) \right], \quad E = \sqrt{\vec{p}^2 + m^2} \\
&= \frac{(m - M_0)^2}{4G_S} - \frac{3}{2\pi^2} \left\{ \left[\Lambda(\Lambda^2 + m^2)^{3/2} - \frac{m^2}{2} \left(\Lambda\sqrt{\Lambda^2 + m^2} + m^2 \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} \right) \right. \right. \\
&\qquad\qquad \left. - (m \to M_0) \right] \\
&\quad + \theta(\mu - m) \left[\frac{\mu}{3}\sqrt{\mu^2 - m^2}(\mu^2 - 4m^2) + \frac{m^2}{2} \left(\mu\sqrt{\mu^2 - m^2} + m^2 \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m} \right) \right] \right\},
\end{aligned}
\tag{2}
$$

where m is the constituent quark mass i.e. the order parameter indicating chiral symmetry breaking, $\mu = \mu_u = \mu_d$ is the common chemical potential of the two flavor quarks and Λ is

the 3D momentum cutoff of the loop integrals. We will focus on the least value points of the effective potential $V(m, \mu)$ which correspond to the ground states of the model. In all the following calculations, Λ and the four-fermion coupling constant G_S will be fixed by the formula of the π decay constant [9]

$$
\begin{aligned}
f_\pi^2 &= -i\, 12m^2 \int \frac{d^4p}{(2\pi)^4} \frac{\theta(\Lambda^2 - \vec{p}^2)}{(p^2 - m^2 + i\varepsilon)^2} \\
&= \frac{3m^2}{2\pi^2} \left(\ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} - \frac{\Lambda}{\sqrt{\Lambda^2 + m^2}} \right),
\end{aligned}
\tag{3}
$$

Gell-Mann Oakes Renner relation [10]

$$
m_\pi^2 = -\frac{M_0 \langle \bar{q}q \rangle}{f_\pi^2}
\tag{4}
$$

and the vacuum form of the following gap equation from the extreme value condition $\partial V(m, \mu)/\partial m = 0$ expressed by

$$
\begin{aligned}
m &= M_0 + \frac{6G_S}{\pi^2} m \left[\Lambda\sqrt{\Lambda^2 + m^2} - m^2 \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} \right. \\
&\quad \left. -\theta(\mu - m) \left(\mu\sqrt{\mu^2 - m^2} - m^2 \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m} \right) \right].
\end{aligned}
\tag{5}
$$

By inputting the experimental value m_π=139.57 MeV [11], the phenomenological value f_π=92.4 MeV [12] and taking the current quark mass

$$
M_0 = 5.55 \text{ MeV},
\tag{6}
$$

we will obtain from Eqs.(3),(4) and the equation (5) with $\mu = 0$ (vacuum)

$$
\Lambda = 636.944 \text{ MeV}, \quad G_S = 5.2866 \times 10^{-6} \text{ MeV}^{-2}.
\tag{7}
$$

In the meantime we also get $m = m_1^0 = 322.39$ MeV which is the only minimum point of $V(m, \mu)$ at $\mu = 0$, since it is easy to verify that the second derivation of $V(m, \mu)$ over m

$$
\begin{aligned}
\frac{\partial^2 V}{\partial m^2} &= \frac{1}{2G_S} + \frac{3}{\pi^2} \left[-\frac{\Lambda^3}{\sqrt{\Lambda^2 + m^2}} + 3\,m^2 \left(\ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} - \frac{\Lambda}{\sqrt{\Lambda^2 + m^2}} \right) \right. \\
&\quad \left. +\theta(\mu - m) \left(\mu\sqrt{\mu^2 - m^2} - 3\,m^2 \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m} \right) \right]
\end{aligned}
\tag{8}
$$

satisfies

$$
\left. \frac{\partial^2 V}{\partial m^2} \right|_{\mu=0,\, m=m_1^0} = 35779.3 \text{ MeV}^2 > 0.
$$

This means (both explicit and spontaneous) chiral symmetry breaking in vacuum. When we increase μ, the extreme value points of $V(m, \mu)$ will be determined by Eqs.(5) and (8). The results have been shown in Table 1. Based on them we may discuss the chiral phase transitions of the model at $T = 0$ and a finite μ.

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Table 1 Variations of the extreme value points of $V(m,\mu)$ as rising of the quark chemical potential μ. Denotations $(V2)_i$ and V_i respectively represent the values of $\partial^2 V/\partial m^2$ and V at the extreme value points m_i^μ $(i=1,2,3)$.

μ (MeV)	0	300	337	339.22 (μ_{c1})	340	350	500	636.97 (μ_{c2})
m_1^μ (MeV)	322.39	322.39	308.56	301.47	297.85			
$(V2)_1$ (MeV^2)	>0	>0	>0	>0	>0			
$V_1/10^8$ (MeV^4)			-6.901	-6.915	-6.921			
m_2^μ (MeV)			174.11	211.71	223.10			
$(V2)_2$ (MeV^2)			<0	<0	<0			
$V_2/10^8$ (MeV^4)			-6.763	-6.858	-6.886			
m_3^μ (MeV)			124.17	102.37	97.62	65.78	11.12	5.55 (M_0)
$(V2)_3$ (MeV^2)			>0	>0	>0	>0	>0	>0
$V_3/10^8$ (MeV^4)			-6.768	-6.915	-6.969	-7.736	-31.66	-83.38

First we note that, if we increase μ but keep $\mu < m_1^0 = 322.39$ MeV, for instance, $\mu = 300$ MeV, then $V(m,\mu)$ will have the only minimum point still at $m = m_1^0$, as shown in the left-third column. This indicates that the chiral symmetry breaking in vacuum will be maintained. However, if μ goes up to above m_1^0, then the situation will be changed. In the left-fourth column of Table 1 we give an example of $\mu = 337$ MeV $> m_1^0$. In this case, the gap equation (5) will have three solutions corresponding to two minimum points $m = m_1^\mu = 308.56$ MeV, $m_3^\mu = 124.17$ MeV and a maximal point $m = m_2^\mu = 174.11$ MeV.

Obviously , since $V(m_1^\mu) < V(m_3^\mu)$, $m_1^\mu = 308.56$ MeV will still be the least value point of $V(m,\mu)$, though it has been less than $m_1^0 = 322.39$ MeV in vacuum. It is found that as μ increases further, $V(m_1^\mu) - V(m_3^\mu)$ will go up, and finally it will occur that $V(m_1^\mu) > V(m_3^\mu)$ so that the least value point of $V(m,\mu)$ transfers from m_1^μ to m_3^μ discontinuously, as shown in the $\mu = 340$ MeV column of Table 1 and this indicates that a first order phase transition has happened. The critical chemical potential μ_{c1} may be determined by the condition

$$V(m_1^{\mu_{c1}}, \mu_{c1}) = V(m_3^{\mu_{c1}}, \mu_{c1})$$

where $m_1^{\mu_{c1}}$ and $m_3^{\mu_{c1}}$ are respectively the two solutions of Eq.(5) with $\partial^2 V/\partial m^2 > 0$. From this we obtain $\mu_{c1} = 339.22$ MeV with $m_1^{\mu_{c1}} = 301.47$ MeV and $m_3^{\mu_{c1}} = 102.37$ MeV, as shown in the left-fifth column of Table 1. It is emphasized that after the the first order phase transition, we have the least value point $m_3^\mu \gg M_0$ and this implies that the quark-antiquark condensates $\langle \bar{q}q \rangle$ remain to have quite large contribution to m, hence the dynamical sector of chiral symmetry breaking, to quite large extent, has still not been restored. As a comparison, we note that in the chiral limit ($M_0 = 0$), we always have the V' corresponding second minimal point $m_3^\mu = 0$ [7], hence the present $m_3^\mu \gg M_0$ can only be attributed to the current quark mass effect. Of course, due to existence of M_0, we could at most have $m_3^\mu = M_0$ after a first order phase transition. Assume that is the case, then the situation will be a little similar to the chiral limit case, i.e. through a first order phase

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transition, the sector of the dynamical chiral symmetry breaking will be restored completely, though the explicit chiral symmetry breaking indicated by M_0 remains. Then the least value point $m_3^\mu = M_0$ will no longer change as a further increase of μ. The phase transition will cease at the first order critical value $\mu = \mu_{c1}$. However, the result $m_3^\mu \gg M_0$ has obviously negated the above assumption.

In fact, as μ continues to increases, the least value point m_3^μ of $V(m, \mu)$ will smoothly decrease and at above some value of μ, it will become the only minimum point of $V(m, \mu)$ left, as shown in the right-third column. A direct question is whether it may be expected that as μ grows up further to some critical value μ_{c2}, we will have $m_3^\mu \to M_0$, thus the dynamical chiral symmetry breaking will be restored completely. To answer this question, we may take $m = M_0$ in Eq.(5) and obtain the equation to determine μ_{c2}

$$M_0^2 \ln \frac{\Lambda + \sqrt{\Lambda^2 + M_0^2}}{\mu + \sqrt{\mu^2 - M_0^2}} = \Lambda \sqrt{\Lambda^2 + M_0^2} - \mu \sqrt{\mu^2 - M_0^2} \tag{9}$$

whose solution is

$$\mu = \mu_{c2} = \sqrt{\Lambda^2 + M_0^2} = 636.97 \text{ MeV}, \tag{10}$$

in view of Eqs.(6) and (7), as is shown in the right-first column. However, it is noted that the resulting "second order" critical chemical potential μ_{c2} has exceeded the 3D momentum cut off Λ which should be considered as the reasonable largest mass scale in this NJL model and is nonphysical. The above result shows that when the current quark mass M_0 exists, within the frame of the NJL model where $\mu < \Lambda$, the limit $m \to M_0$ or $\langle \bar{q}q \rangle = 0$ is not realizable. This fact implies, on the one hand, that the dynamical chiral symmetry breaking can not be restored completely; and on the other hand, that after a first order phase transition one will get only a smooth crossover behavior of the order parameter m which, however, can never arrive at the limit value M_0.

3   ZERO T AND HIGH μ PHASE TRANSITIONS WITH ELECTRICAL NEUTRALITY

In the above discussions we did not consider the electrical neutrality (EN) condition of the quark matter. In this section we will extend our discussions to the case with EN condition and examine if similar conclusion can be reached. It is noted that the quark matter with EN is a more realistic case where electron must be included in the chemical equilibrium of weak decays of the quarks. In the electric neutrality case, the effective potential $V(m, \mu, \mu_e)$ in the mean field approximation can be expressed by [8]

$$V(m, \mu, \mu_e)$$

$$= \frac{(m - M_0)^2}{4G_S} - 6 \int \frac{d^3 p}{(2\pi)^3} \left[2(E - \sqrt{\vec{p}^2 + M_0^2}) + \theta(\mu_u - E)(\mu_u - E) \right.$$

$$\left. + \theta(\mu_d - E)(\mu_d - E) \right] - \frac{\mu_e^4}{12\pi^2}, \quad E = \sqrt{\vec{p}^2 + m^2}$$

$$= \frac{(m - M_0)^2}{4G_S} - \frac{3}{4\pi^2} \left\{ \left[2\Lambda(\Lambda^2 + m^2)^{3/2} - m^2 \left(\Lambda \sqrt{\Lambda^2 + m^2} + m^2 \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} \right) \right. \right.$$

$$\left. - (m \to M_0) \right]$$

$$+ \left(\theta(\mu_u - m) \left[\frac{\mu_u \sqrt{\mu_u^2 - m^2}}{3} (\mu_u^2 - 4m^2) + \frac{m^2}{2} \left(\mu_u \sqrt{\mu_u^2 - m^2} \right. \right. \right.$$

$$\left. \left. \left. + m^2 \ln \frac{\mu_u + \sqrt{\mu_u^2 - m^2}}{m} \right） \right] + (\mu_u \to \mu_d) \right) \right\} - \frac{\mu_e^4}{12\pi^2}, \quad (11)$$

where $\mu = -\partial V / \partial n$ is the quark chemical potential corresponding to the total quark number density n, μ_e is the chemical potential of electron and

$$\mu_u = \mu - \frac{2}{3} \mu_e, \quad \mu_d = \mu + \frac{1}{3} \mu_e = \mu_u + \mu_e, \quad (12)$$

are respectively the chemical potential of the u, d quarks. The last equality of Eq.(12) is usually referred as beta equilibrium [9]

For deriving the electrical neutrality condition, it is noted that the electrical charge density in the two-flavor quark matter with electrons is

$$n_Q = \frac{2}{3} n_u - \frac{1}{3} n_d - n_e$$

with n_u, n_d and n_e denoting respectively the number density of the u, d quark and electron. From it we may obtain

$$\mu_e = -\frac{\partial V}{\partial n_e} = -\frac{\partial V}{\partial n_Q} \frac{\partial n_Q}{\partial n_e} = -\mu_Q.$$

Hence the EN condition will become $n_Q = -\partial V / \partial \mu_Q = \partial V / \partial \mu_e = 0$ and has the following explicit expression

$$\frac{\partial V}{\partial \mu_e} = \frac{1}{3\pi^2} \left\{ 2\theta(\mu - \frac{2\mu_e}{3} - m) \left[(\mu - \frac{2\mu_e}{3})^2 - m^2 \right]^{3/2} \right.$$

$$\left. - \theta(\mu + \frac{\mu_e}{3} - m) \left[(\mu + \frac{\mu_e}{3})^2 - m^2 \right]^{3/2} - \mu_e^3 \right\} = 0. \quad (13)$$

Owing to Eq.(13), for a given μ, we will have $\mu_e = \mu_e(m)$. Hence, in view of the EN condition $\partial V / \partial \mu_e = 0$, the extreme value condition of the effective potential $V(m, \mu, \mu_e)$ now becomes

$$\frac{dV}{dm} = \frac{\partial V}{\partial m} + \frac{\partial V}{\partial \mu_e} \frac{\partial \mu_e}{\partial m} = \frac{\partial V}{\partial m} = 0,$$

then from the first equality of Eq.(11), it is easy to obtain the gap equation $\partial V/\partial m = 0$ with the explicit form

$$
\begin{aligned}
m \;=\; & M_0 + \frac{3G_S}{\pi^2}\, m \left\{ 2 \left(\Lambda\sqrt{\Lambda^2 + m^2} - m^2 \ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} \right) \right. \\
& \left. - \left[\theta(\mu_u - m) \left(\mu_u \sqrt{\mu_u^2 - m^2} - m^2 \ln \frac{\mu_u + \sqrt{\mu_u^2 - m^2}}{m} \right) + (\mu_u \to \mu_d) \right] \right\}. \; (14)
\end{aligned}
$$

Owing to $dV/dm = \partial V/\partial m$ in the EN condition, the second derivative of $V(m, \mu, \mu_e)$ over m may be expressed by

$$
\begin{aligned}
\frac{d^2 V}{dm^2} \;=\; & \frac{\partial}{\partial m}\left(\frac{dV}{dm}\right) + \frac{\partial}{\partial \mu_e}\left(\frac{dV}{dm}\right)\frac{\partial \mu_e}{\partial m} \\
\;=\; & \frac{\partial^2 V}{\partial m^2} + \frac{\partial^2 V}{\partial \mu_e \partial m}\frac{\partial \mu_e}{\partial m}.
\end{aligned}
$$

From the EN condition $\partial V/\partial \mu_e = 0$ we may have

$$
\frac{d}{dm}\left(\frac{\partial V}{\partial \mu_e}\right) = \frac{\partial^2 V}{\partial m \partial \mu_e} + \frac{\partial^2 V}{\partial \mu_e^2}\frac{\partial \mu_e}{\partial m} = 0\,.
$$

It leads to

$$
\frac{\partial \mu_e}{\partial m} = -\frac{\partial^2 V}{\partial m \partial \mu_e}\bigg/\frac{\partial^2 V}{\partial \mu_e^2}\,.
$$

Hence the second derivative of $V(m, \mu, \mu_e)$ over m becomes

$$
\begin{aligned}
\frac{d^2 V}{dm^2} \;=\; & \frac{\partial^2 V}{\partial m^2} - \left(\frac{\partial^2 V}{\partial m \partial \mu_e}\right)^2 \bigg/ \frac{\partial^2 V}{\partial \mu_e^2} \\
\;=\; & \frac{1}{2G_S} + \frac{3}{\pi^2}\left\{ 3m^2 \left(\ln \frac{\Lambda + \sqrt{\Lambda^2 + m^2}}{m} - \frac{\Lambda}{\sqrt{\Lambda^2 + m^2}} \right) - \frac{\Lambda^3}{\sqrt{\Lambda^2 + m^2}} \right. \\
& + \frac{1}{2}\left[\theta(\mu_u - m)\left(\mu_u \sqrt{\mu_u^2 - m^2} - 3m^2 \ln \frac{\mu_u + \sqrt{\mu_u^2 - m^2}}{m} \right) + (\mu_u \to \mu_d) \right] \\
& \left. + \frac{m^2 \left[2\theta(\mu_u - m)\sqrt{\mu_u^2 - m^2} - \theta(\mu_d - m)\sqrt{\mu_d^2 - m^2} \right]^2}{4\theta(\mu_u - m)\mu_u\sqrt{\mu_u^2 - m^2} + \theta(\mu_d - m)\mu_d\sqrt{\mu_d^2 - m^2} + 3\mu_e^2} \right\}. \quad (15)
\end{aligned}
$$

Now for a given μ, the extreme value points of $V(m, \mu, \mu_e)$ will be determined by the simultaneous equations (13) and (14), and except for this, the whole discussions of the chiral phase transitions under EN condition will be parallel to the ones made in Sect.2. In Table 2 we list the variations of the extreme value points of $V(m, \mu, \mu_e)$ for some selected values of μ in a successively increasing order.

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Table 2 Variations of the extreme value points of $V(m, \mu, \mu_e)$ as rising of μ under electrical neutrality condition. Denotations μ_{ei}, $(dV2)_i$ and V_i ($i = 1, 2, 3$) represent respectively the values of μ_e, d^2V/dm^2 and V at the extreme value points m_i^μ.

μ (MeV)	0	300	345	345.6 (μ_{c1})	346	350	500	656.87 (μ_{c2})
m_1^μ (MeV)	322.39	322.39	283.69	277.81	272.27			
μ_{e1} (MeV)	0	0	23.89	26.18	28.23			
$(dV2)_1$ (MeV^2)	> 0	> 0	> 0	> 0	> 0			
$V_1/10^8$ (MeV^4)			-6.958	-6.967	-6.974			
m_2^μ (MeV)			189.12	209.32	221.93			
μ_{e2} (MeV)			52.31	47.32	43.97			
$(dV2)_2$ (MeV^2)			< 0	< 0	< 0			
$V_2/10^8$ (MeV^4)			-6.928	-6.953	-6.968			
m_3^μ (MeV)			143.58	130.74	125.3	96.37	12.3	5.55 (M_0)
μ_{e3} (MeV)			62.28	64.53	65.54	70.65	109.41	143.81
$(dV2)_3$ (MeV^2)			> 0	> 0	> 0	> 0	> 0	> 0
$V_3/10^8$ (MeV^4)			-6.931	-6.967	-6.992			

The left-second and left-third column of Table 2 indicate that, and as has been actually checked, the chiral symmetry breaking in vacuum will be maintained up to $\mu \leq m_1^0 = 322.39$ MeV. If $\mu = 345\ MeV > m_1^0$ in vacuum, then the effective potential $V(m, \mu, \mu_e)$ will show two minimal points m_1^μ and m_3^μ and a maximal point m_2^μ. It may be seen that in this case $V_1 < V_3$ and this merely implies that the least value point m_1^μ of $V(m, \mu, \mu_e)$ in vacuum smoothly changes to a lower value and no phase transition happens. However, if $\mu = 346$ MeV, we will have $V_1 > V_3$. This indicates that the ground state has been transferred discontinuously to the minimal point $(m_3^\mu = 125.3$ MeV, $\mu_{e3} = 65.54$ MeV) and a first phase transition must have happened. The critical chemical potential $\mu = \mu_{c1}^{EN}$ should be determined by the equation

$$V(m_1^\mu, \mu, \mu_{e1}) = V(m_3^\mu, \mu, \mu_{e3}),$$

where (m_1^μ, μ_{e1}) and (m_3^μ, μ_{e3}) are respectively the two minimal points of $V(m, \mu, \mu_e)$. The result $\mu = \mu_{c1}^{EN} = 345.6$ MeV is listed in the left-fifth column of Table 2.

At this point, the smaller minimal value point $m_3^{\mu_{c1}^{EN}} = 130.74$ MeV $\gg M_0$, this again implies that after the first order phase transition the chiral condensates $\langle \bar{q}q \rangle$ have not been reduced to zeros, thus the dynamical chiral symmetry breaking has not restored completely. It is seen from Table 2 that as μ continue to go up, for example, for $\mu > 350$ MeV, the effective potential $V(m, \mu, \mu_e)$ will have the only minimal point m_3^μ left whose values will decrease

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gradually. Finally mathematically $m_3^\mu = M_0$ can be arrived at $\mu = \mu_{c2}^{EN} = 656.87\,\text{MeV}$ and $\mu_e = \mu_{e3} = 143.81\,\text{MeV}$ which are solutions of Eqs.(13) and (14) with m replaced by M_0, as shown in the left-last column of Table 2. The smooth change of the quark mass m_3^μ to its current mass M_0 indicates that complete restoration of the dynamical chiral symmetry breaking, if it is realizable, seems to be a "second order phase transition". However, it is again found that the "critical chemical potential" μ_{c2}^{EN} is higher than the reasonable largest mass scale Λ of the NJL model thus the condition is nonphysical. In addition, it also demands a quite large and unrealistic electrical chemical potential μ_e.

A comparison between the results with and without EN condition is shown in Table 3.

Table 3 Comparison between the results with and without EN condition.

	μ_{c1} (MeV)	$m_3^{\mu_{c1}}$ (MeV)	μ_{c2} (MeV)
EN	345.6	130.74	656.87
non-EN	339.22	102.37	636.97

It is seen that, in the case with EN condition, the first order critical quark chemical potential μ_{c1} and the smaller least value point $m_3^{\mu_{c1}}$ of $V(m, \mu, \mu_e)$ at the critical point are bigger than the corresponding ones in the case without EN condition. The former result is consistent with the conclusion derived in Ref.[8] that in the EN condition, a first order phase transition must happen at a larger value of μ_u or $\mu = \mu_u + (2/3)\mu_e$ than the one in the non-EN condition. In addition, with EN, the "second order critical chemical potential" μ_{c2} to achieve $m \to M_0$ is higher than the one without EN, thus it even more exceeds the largest mass scale Λ of the model.

In short, inclusion of the EN condition does not change the qualitative behavior of chiral phase transitions of the model with the current quark mass M_0. Not only after a first order phase transition but also in the whole variations of m, the chiral condensates $\langle \bar{q}q \rangle$ are always not equal to zeros within the frame of the NJL model where $\mu < \Lambda$. Therefore, we can not have complete restoration of the dynamical chiral symmetry breaking and finally, instead of talking about a second order phase transition, only get a crossover behavior of the order parameter m as μ increases.

4   A GENERAL ANALYSIS FOR ANY T AND μ CASE

In this section, we will make a general analysis of the chiral phase transitions of the model in any T and μ case. For simplicity and without loss of generality, we will consider only the case without EN condition. The effective potential $V(m, T, \mu)$ at finite T and μ in the mean field approximation may be expressed by [9]

$$
\begin{aligned}
V(m, T, \mu) &= \frac{(m - M_0)^2}{4G_S} - 12 \int \frac{d^3p}{(2\pi)^3} \left\{ E - \sqrt{\vec{p}^2 + M_0^2} \right. \\
&\left. + T \ln \left[1 + e^{-(E-\mu)/T} \right] + T \ln \left[1 + e^{-(E+\mu)/T} \right] \right\}, \quad E = \sqrt{\vec{p}^2 + m^2}. \quad (16)
\end{aligned}
$$

The gap equation $\partial V(m, T, \mu)/\partial m = 0$ becomes

$$
\begin{aligned}
m &= M_0 + \frac{12 G_S}{\pi^2} m \\
&\times \int_0^\Lambda dp \frac{p^2}{E} \left[1 - \frac{1}{e^{(E-\mu)/T} + 1} - \frac{1}{e^{(E+\mu)/T} + 1} \right]. \quad (17)
\end{aligned}
$$

The second derivative of $V(m, T, \mu)$ over m may be expressed by

$$\frac{\partial^2 V}{\partial m^2} = \frac{1}{2\,G_S} - \frac{6}{\pi^2} \int_0^\Lambda dp\, \frac{p^4}{E^3} \left[1 - \frac{1}{e^{(E-\mu)/T} + 1} - \frac{1}{e^{(E+\mu)/T} + 1} \right]$$
$$- \frac{6\,m^2}{\pi^2 T} \int_0^\Lambda dp\, \frac{p^2}{E} \left\{ \frac{e^{(E-\mu)/T}}{\left[e^{(E-\mu)/T} + 1 \right]^2} + (\mu \to -\mu) \right\}. \tag{18}$$

By means of Eqs.(17) and (18) and the parameters given by Eqs.(6) and (7), we may find out the least value points of $V(m, T, \mu)$. For a given low T, analogous high μ chiral transitions to the ones at zero T as described in Sects. 2 and 3 will also appear and the discussions may be conducted similarly. Therefore, in Table 4 we list only the least value points of $V(m, T, \mu)$ at the critical chemical potential μ if a first order phase transition could happen at a given low T, or otherwise, the only minimal (least) value point of $V(m, T, \mu)$ at a higher T.

Table 4 Variations of the least value points of $V(m, T, \mu)$ as change of T and μ.
For a given T and μ, m_1 and m_3 represent the critical least value points with $V(m_1) = V(m_3)$ and m_2 is the corresponding maximal value point. $(V2)_i$ and V_i $(i = 1, 2, 3)$ represent respectively the values of $\partial^2 V/\partial m^2$ and V at the points m_i.

T (MeV)	0	20	35	37.1	37.5	50	100	100
μ (MeV)	339.22	335.52	328.48	327.27	327.03	327	327	500
m_1 (MeV)	301.47	277.89	219.93	191.35	186.23	95.47	44.81	11.17
$(V2)_1$ (MeV^2)	6920.41	4419.43	493.8	7.287	> 0	> 0	> 0	> 0
$V_1/10^8$ (MeV^4)	-6.915	-7.100	-7.5222	-7.605				
m_2 (MeV)	211.71	203.17	189.32	187.58				
$(V2)_2$ (MeV^2)	-2332.48	-1591.39	-220.27	-4.605				
$V_2/10^8$ (MeV^4)	-6.858	-7.074	-7.5217	-7.605				
m_3 (MeV)	102.37	114.53	156.79	182.63				
$(V2)_3$ (MeV^2)	3042.59	2211.86	387.72	9.19				
$V_3/10^8$ (MeV^4)	-6.915	-7.100	-7.5222	-7.605				

It has been verified that at a given sufficiently low temperature T, as μ increases, we will first have a first order phase transition and at the first order critical quark chemical potential μ, we will always have the least value points of $V(m, T, \mu)$ $m_1 > m_3 \gg M_0$, as shown in the left-first columns of Table 4. This implies again that after the first order phase transitions we have not achieved the condensates $\langle \bar{q}q \rangle = 0$ and the complete restoration of the dynamical chiral symmetry breaking, similar to the case of $T = 0$.

On the other hand, it is noted that as rising of T, for instance, from 0 to 37.1 MeV,

besides that the first order critical quark chemical potential μ will slightly go down (from 339.22 MeV to 327.27 MeV), at the first order critical quark chemical potential μ, the bigger minimal point m_1 will go down (from 301.47 MeV down to 191.35 MeV) and the smaller minimal point m_3 will go up (from 102.37 MeV up to 182.63 MeV). This shows that the interval between m_1 and m_3 will become smaller and smaller. In the meantime, the curvature ($\propto |\partial^2 V/\partial m^2|$) at the minimal points m_1 and m_3 (and also including at the maximal point m_2) will also become smaller and smaller. The final result will be that the two minimal points m_1 and m_3 will merge into a single one so that $V(m, T, \mu)$ will have the mere minimal point m_1 left. This indicates that the first order phase transitions will end at $T = 37.1$ MeV and $\mu = 327.27$ MeV. Once $T > 37.1$ MeV, the effective potential $V(m, T, \mu)$ will always have the only minimal (also least value) point left whose location m_1 will smoothly decrease as a further increase of T and μ, as shown in Table 4. As a result, in the $T - \mu$ phase diagram of the model, $T = 37.1$ MeV and $\mu = 327.27$ MeV will become a critical end pint i.e. at this point the first order phase transition line ends.

It should be emphasized that for the appearance of the critical end point, a decisive factor is the nonzero-ness of the chiral condensates $\langle \bar{q}q \rangle$ i.e. $m_3 \gg M_0$ after a first order phase transition. It is the nonzero-ness of $\langle \bar{q}q \rangle$ that makes it possible that the two minimal points m_1 and m_3 could be moved and finally merge into a single one. Otherwise, if it is assumed that we may have $m_3 \to$ the fixed M_0 i.e. $\langle \bar{q}q \rangle \to 0$ after a first order phase transition, and furthermore, the only minimal point m_1 of $V(m, T, \mu)$ at a higher T can finally be reduced to M_0 (similar to a variation of the order parameter in a second order phase transition), then we will obtain, similar to the chiral limit case with $M_0 = 0$, a tricritical point, rather than a critical end point.

Of course, if we have $m_3 \gg M_0$ after a first order phase transition but assume that as a further increase of T and/or μ, both m_3 and the least value point m_1 of $V(m, T, \mu)$ at a higher T could be reduced to M_0, then besides that the explicit chiral symmetry breaking is always kept in the phase transitions and the critical end point remains to exist, we will also get a "second order" phase transition line. However, we will prove that this situation can not practically happen within the frame of the NJL model.

As is shown in Table 4, for the least value points m of $V(m, T, \mu)$, we always have $m = m_3 > M_0$ after a first order phase transition or $m = m_1 > M_0$ at a higher T. In addition, similar to that indicated in the right-last columns of Table 4, m will decrease as a further increase of T and μ. Thus a natural question is that if at some high value of T and μ, the limit $m \to M_0$, corresponding to total disappearance of the chiral condensates or equivalently, complete restoration of the dynamical chiral symmetry breaking, could be achieved within the frame of the NJL model. For giving the answer of this question, we must solve the gap equation (17) with m replaced by M_0. In this case Eq.(17) becomes

$$\int_0^\Lambda dp \frac{p^2}{E_0} \left[1 - \frac{1}{e^{(E_0 - \mu)/T} + 1} - \frac{1}{e^{(E_0 + \mu)/T} + 1} \right] = 0,$$

$$E_0 = \sqrt{p^2 + M_0^2} \tag{19}$$

which contains only the parameters Λ and M_0, independent of G_S. The problem is reduced to find out the solutions of Eq.(19) about T and μ.

First for a given T, consider possible values of μ.

We begin with $T = 0$. When $T \to 0$, we have $1/[e^{(E_0 + \mu)/T} + 1] \to 0$, thus Eq.(19) will be reduced to Eq.(9) which has the solution $\mu = \sqrt{\Lambda^2 + M_0^2}$, as given in Sect.2.

For a finite T, we have numerically solved Eq.(19) and obtained the results listed in Table 5.

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Table 5 Solutions of Eq.(19) for some finite T.

T (MeV)	0	20	50	100	200
μ (MeV)	636.97	1102.02	1858.53	3138.57	5781.7

These data indicate that at these (and believably any) finite T, the resulting values of μ are always bigger or much bigger than Λ, thus are not physically reasonable.

On the other hand, we may consider the solutions of Eq.(19) when $\mu < \sqrt{\Lambda^2 + M_0^2}$. In fact, numerical calculations give $T = 4.06 \times 10^8$ MeV for all $\mu < \sqrt{\Lambda^2 + M_0^2}$. In view of the working precision of the calculations, this merely indicates that $T \to \infty$ is the only solution of Eq.(19) if $\mu < \sqrt{\Lambda^2 + M_0^2}$. However, the result $T \to \infty$ is obviously physically unrealistic.

The above results can be given a simple physical explanation. It is noted that the gap equation (17) at finite T and μ is identical to

$$m = M_0 - 2G_S \langle \bar{q}q \rangle_T, \tag{20}$$

where $\langle \bar{q}q \rangle_T$ is the thermal quark-antiquark condensates which can be obtained from the quark-antiquark condensates $\langle \bar{q}q \rangle$ in vacuum through replacement of the quark propagator by its form in thermal field theory [13].

In fact, the quark-antiquark condensates $\langle \bar{q}q \rangle$ in vacuum can be expressed by

$$\langle \bar{q}q \rangle = N_f N_c \int \frac{d^4 p}{(2\pi)^4} \mathrm{tr} \frac{i}{\slashed{p} - m + i\varepsilon}, \tag{21}$$

where N_f and N_c are the quark's flavor and color number respectively. Taking the real-time formalism of thermal field theory, then in Eq.(21) making the replacement

$$
\begin{aligned}
\frac{i}{\slashed{p} - m + i\varepsilon} &\to \cos^2 \theta_p \frac{i}{\slashed{p} - m + i\varepsilon} + \sin^2 \theta_p \frac{i}{\slashed{p} - m - i\varepsilon}, \\
\sin^2 \theta_p &= \theta(p^0) n(p^0 - \mu) + \theta(-p^0) n(-p^0 + \mu), \\
n(p^0 - \mu) &= \frac{1}{e^{(p^0 - \mu)/T} + 1},
\end{aligned}
\tag{22}
$$

we may express the thermal quark-antiquark condensates by

$$
\begin{aligned}
\langle \bar{q}q \rangle_T &= -4N_f N_c m \int \frac{d^4 p}{(2\pi)^4} \left[\frac{i}{p^2 - m^2 + i\varepsilon} - 2\pi \sin^2 \theta_p \delta(p^2 - m^2) \right] \\
&= -4N_f N_c \int \frac{d^3 p}{(2\pi)^3} \frac{m}{2E} \left[1 - n(E - \mu) - n(E + \mu) \right] \\
&= -\frac{6m}{\pi^2} \int_0^\Lambda dp \, \frac{p^2}{E} \left[1 - \frac{1}{e^{(E-\mu)/T} + 1} - \frac{1}{e^{(E+\mu)/T} + 1} \right],
\end{aligned}
\tag{23}
$$

where $N_f = 2$ and $N_c = 3$ have been taken. In view of Eq.(23), we immediately see that Eq.(20) is just the gap equation (17).

Now it is easy to find the difference between $M_0 = 0$ and $M_0 \neq 0$ for Eq.(20). When $M_0 = 0$, i.e. in the chiral limit, Eq.(20) will become

$$1 = -\frac{2G_S}{m} \langle \bar{q}q \rangle_T.$$

It may have the solution $m = 0$ at some finite and reasonable T and μ which is identical to that the thermal condensates $\langle \bar{q}q \rangle_T = 0$ and indicates a complete restoration of the

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dynamical chiral symmetry breaking. On the other hand, when $M_0 \neq 0$, if one also expects such complete restoration of the dynamical chiral symmetry breaking, then it must have the result that

$$\langle \bar{q}q \rangle_T |_{m=M_0} = 0.$$

However the proceeding discussions in this section just show that when the current quark mass $M_0 \neq 0$ the above result could not be attained at a finite T and a reasonable $\mu < \Lambda$ i.e. within the frame of a NJL model where $\mu, T < \Lambda$. We indicate that it is just a nonzero $\langle \bar{q}q \rangle_T$ that can have a continuative smooth decrease as rising of T and μ thus could lead to the crossover behavior of the order parameter m indicating chiral symmetry breaking at high T and high μ. Otherwise, if $m \to M_0$ i.e. the condensates $\langle \bar{q}q \rangle = 0$ could be achieved finally within the frame of the NJL model, then, as stated above, it will lead to a "second order phase transition line" rather than the crossover behavior of m in the $T - \mu$ phase diagram.

In brief, it is the nonzero-ness of the chiral condensates induced by the current quark mass M_0 which leads to the appearance of the known critical end point and the crossover behavior of the order parameter m in the phase diagram of the NJL model. In view of the simulating feature of the NJL model for QCD, such phase diagram is very well qualitatively consistent with the ones obtained based on real QCD dynamics with the current quark mass [14].

In addition, the condensates $\langle \bar{q}q \rangle \neq 0$ at a finite $T, \mu < \Lambda$ demonstrated here will also force one to consider the interplay between the quark-antiquark condensates $\langle \bar{q}q \rangle$ and the diquark condensates $\langle qq \rangle$ when one researches the color superconductors in zero T and middle μ case, where μ has exceeded the first order critical quark chemical potential μ_{c1} and the diquark condensates could be formed. This point has been noticed and discussed earlier [9, 15], and the analysis made in the present chapter certainly strengthen theoretical grounds of the above consideration.

5 CONCLUSION

In this chapter, by means of a quantitative investigation of the order parameters in the ground states we have reexamined and analyzed systematically the effect of the current quark mass M_0 on nonzero-ness of the chiral condensates $\langle \bar{q}q \rangle$ in chiral phase transitions of a two-flavor NJL model simulating QCD respectively in the cases of zero T and high μ without and with the electrical neutrality condition as well as in the case of any T and μ without the electrical neutrality condition. The nonzero-ness of the chiral condensates $\langle \bar{q}q \rangle$ induced by the current quark mass M_0 is found to have a double meanings: one is that the condensates $\langle \bar{q}q \rangle$ keep to have quite large values after a first order phase transition at a low T and a large $\mu = \mu_{c1}$; the other one is that although at a high T or a large $\mu > \mu_{c1}$, the chiral condensates $\langle \bar{q}q \rangle$ will smoothly decrease as a further rising of T and/or μ, they can never be reduced to zeros within the frame of the NJL model, i.e. in the condition when $\mu, T < \Lambda$, where the momentum cutoff Λ of the loop integrals must be viewed the largest physical mass scale of the NJL model.

For the latter one, in fact, the mathematical solving of the gap equation indicates that in the case of $T = 0$, the order parameter $m \to M_0$ i.e. the chiral condensates $\langle \bar{q}q \rangle = 0$ could be achieved only at $\mu = \sqrt{\Lambda^2 + M_0^2}$ if the EN condition is not imposed, and even at a higher μ if the EN condition is imposed. In the general case, it has been proven that when the current quark mass $M_0 \neq 0$, the limit $m \to M_0$ can be attained only in the conditions that $\mu \geq$ or $\gg \sqrt{\Lambda^2 + M_0^2}$ if $T = 0$ and finite, or $T \to \infty$ if $\mu < \sqrt{\Lambda^2 + M_0^2}$. Obviously, these conditions are all non-physical and unrealistic.

The above results indicate that once the current quark mass M_0 exists, it is impossible

to achieve complete restoration of the dynamical chiral symmetry breaking within the frame of the NJL model. This includes both after a first order phase transition and in the whole process of chiral phase transitions. Theoretically this reflects a close and deep interconnection between the explicit and the dynamical (spontaneous) chiral symmetry breaking.

The nonzero-ness of the chiral condensates $\langle \bar{q}q \rangle$ plays a decisive role in the structure of the phase diagram of the NJL model with the current quark mass M_0. It is just the nonzero-ness that leads to the appearance of a critical end point rather than a tricritical point, and a crossover behavior of the order parameter at high T and μ rather than a second order phase transition line in the known phase diagram of the NJL model. In addition, it also gives a strong theoretical grounds for the point of view that one must consider the interplay between the quark-antiquark condensates $\langle \bar{q}q \rangle$ and the diquark condensates $\langle qq \rangle$ when researching the color superconductor at zero T and middle μ.

To sum up, the analysis made in present chapter of the nonzero-ness of the chiral condensates induced by current quark mass and its physical effects may certainly deepen our theoretical understanding of the mechanism of chiral phase transitions of the NJL model. It is also an excellent example from which one can understand how a source term of the Lagrangian can greatly affect dynamical behavior of a physical system.

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